CPIUC- Manual
Versión 2.0

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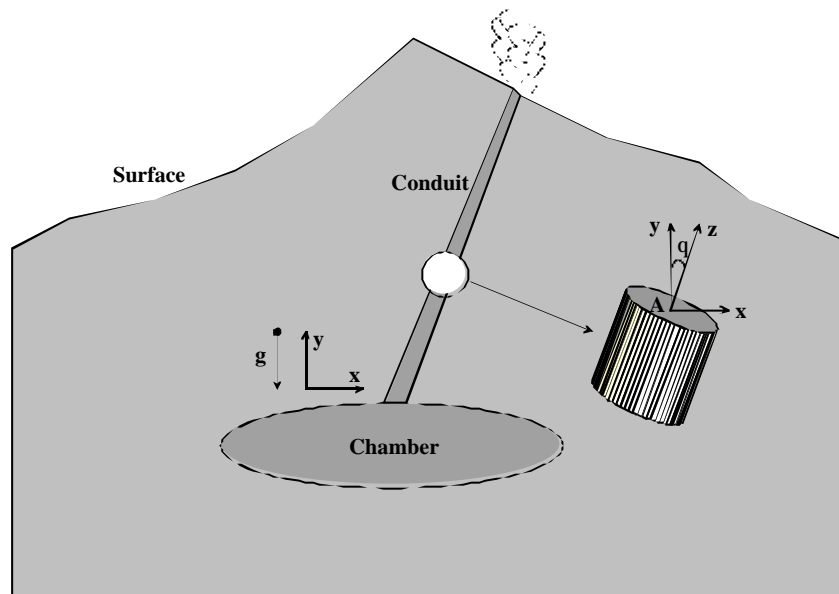
Please, send any comment or suggestion to any of the authors.

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1. Introduction

CPIUC is a FORTRAN code designed to estimate the temporal evolution of the main eruptive parameters during an explosive eruption. The name CPIUC is the acronym for "**C**hamber **PIU** (the Italian for plus) **C**onduit". The generic domain is composed by a magmatic reservoir connected to the Earth surface trough a volcanic conduit. Both the magma chamber and the volcanic conduit can have arbitrary geometries that can be specified by the user. The flow within the conduit is parallel to the z-direction, which does not necessarily coincide with the vertical y-direction (i.e. conduit inclination can be contemplated).

This manual is structured as follows. Chapter 2 describes the assumptions and the hypothesis behind the physical model implemented. The different closure and constitutive equations contemplated in this version are exposed and the limitations of the code are discussed. Chapter 3 concerns to the execution of the code. It describes the installation and the input and output files. Finally, Appendix A describes classical aspects of the Poiseulle theory necessary to justify the modelling of viscous effects in the conduit.

2. Physical model

2.1. Magma chamber model

The basic assumptions concerning the magma chamber emptying model are:

• Pressure distribution inside the chamber is **magmastatic** [Martí et al., 2000] and, in consequence, it varies only along the vertical coordinate y. Such an approximation can be justified on the basis of more complex numerical results of magma chamber thermo-fluid-

dynamics that show how the gravitational body force controls the pressure distribution inside the reservoir [Folch et al., 1998].

• **Constant temperature**. The density of the magmatic mixture depends only on pressure (barotropic state law).

• The **volatile mass fraction** is constant of varies linearly with depth.

Under these hypothesis, pressure and mass distributions inside the chamber are given by:

$$\frac{dp}{dy} = \boldsymbol{r}(p)\left[\vec{\mathbf{y}} \cdot \vec{\mathbf{g}}\right] = -\boldsymbol{r}(p)\, g_y \tag{1.a}$$

$$\frac{dM}{dy} = \boldsymbol{r}(p)\, A(y) \tag{1.b}$$

where p is pressure, $\boldsymbol{r}$ is the density of the mixture, $\vec{\mathbf{g}}$ is the gravity vector (assumed to point downwards in the y direction), $\vec{\mathbf{y}}$ is the unitary vector in the vertical direction, M is the mass and A is the chamber section at a given depth y. Given the value of pressure at the chamber top, the above equations can be integrated to obtain pressure and mass distributions inside the chamber. The pressure at the chamber top represents also the boundary condition for the conduit model (see section 2.2) that gives the mass flow-rate $\dot{m}_c$ as a function of the pressure at the base of the conduit. In consequence, the conduit model allows to express the temporal variation of M as a function of $\dot{m}_c$ as:

$$M = M_o - \int_0^t \dot{m}_c\, dt \tag{2}$$

where M_o is the mass initially present in the chamber and t is the time lapsed from the beginning of the eruption. The knowledge of pressure at the chamber top defines indeed the time evolution of the pressure distribution in the chamber plus conduit system, $p = p(y,t)$, which represents a very important information to describe the evolution of the physical conditions at the vent and the syn-eruptive deformation of the surrounding country rocks.

2.2. The conduit model

The assumptions of the conduit model are:

• **Steady state**. This is justified by the fact that the time-scale of the pressure variations at conduit base is of the order of hours, much longer than the travel time of the magma in conduit which is of the order of minutes. In other words, given a value of pressure at the chamber top (at the conduit entrance) it is assumed that the flow within the conduit fits "instantaneously" to it.

• **Homogeneous flow**. The gas and the condensed phase (liquid + crystals) have the same velocity both below and above the fragmentation level. Typically, magma enters the conduit in

the bubbly flow regime and exits the conduit in the gas-particle regime, after fragmentation. Fragmentation occurs when the gas volumetric fraction in the mixture, α, reaches a critical value.

• The flow is governed by the **averaged one-dimensional** mass and momentum balance equations. The flow is therefore unidirectional along the z-direction, but the direction of the flow is not restricted to the vertical (inclined conduits can be contemplated). The size of the conduit may vary with depth.

• As in the chamber model, **temperature is constant**. The density of the magmatic mixture depends only on pressure (barotropic state law).

Under the above hypothesis, the equations for the conduit model are:

$$\frac{\partial (\mathbf{r}\, u\, A)}{\partial z} = A\,\hat{c} \tag{3.a}$$

$$\mathbf{r}\, u\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial z} + \mathbf{r}\,[\vec{\mathbf{z}}\cdot\vec{\mathbf{g}}] + F_W \tag{3.b}$$

where u is the averaged velocity (in the flow direction), $A(z)$ is the conduit section, $\hat{c}$ is the mass flow rate due to erosion ($[\hat{c}]$=Kg/m^{3}s), $F_W(u)$ is the wall friction coefficient, and $\vec{\mathbf{z}}$ is the unit vector in the flow direction. Note how, in general, the contribution of the body force term depends on the conduit inclination with respect to the vertical. Defining $a^2 = dp/d\mathbf{r}$ and the Mach number $M = u/a$, equations (3) can be rewritten as:

$$\frac{\partial u}{\partial z} = \frac{u}{(1-M^2)\,a^2}\left[-\vec{\mathbf{z}}\cdot\vec{\mathbf{g}} - \frac{F_W}{\mathbf{r}}\right] + \frac{\hat{c}}{\mathbf{r}\,(1-M^2)} - \frac{u}{(1-M^2)}\frac{1}{A}\frac{dA}{dz} \tag{4.a}$$

$$\frac{\partial p}{\partial z} = -\frac{\mathbf{r}}{(1-M^2)}\left[-\vec{\mathbf{z}}\cdot\vec{\mathbf{g}} - \frac{F_W}{\mathbf{r}}\right] - \frac{u}{(1-M^2)}\hat{c} + \frac{\mathbf{r}\,u^2}{(1-M^2)}\frac{1}{A}\frac{dA}{dz} \tag{4.b}$$

In order to avoid singularities when the Mach approximates to 1, the above equations are normally solved using the Mach number as independent variable. In this case they take the following form:

$$\frac{\partial p}{\partial M} = \frac{-\mathbf{r}\,a^2\left(-\vec{\mathbf{z}}\cdot\vec{\mathbf{g}} - \dfrac{F_W}{\mathbf{r}}\right) - a^2 u\hat{c} + \mathbf{r}\,a^2 u^2\dfrac{1}{A}\dfrac{dA}{dz}}{M\left[\left(1 + a\mathbf{r}\dfrac{da}{d\mathbf{r}}\right)\left(-\vec{\mathbf{z}}\cdot\vec{\mathbf{g}} - \dfrac{F_W}{\mathbf{r}}\right) + \left(1 + \mathbf{r}\,M^2 a\dfrac{da}{dp}\right)\left(\dfrac{a\hat{c}}{M\mathbf{r}} - \dfrac{a^2}{A}\dfrac{dA}{dz}\right)\right]} \tag{5.a}$$

4

$$\frac{\partial z}{\partial M} = \frac{(1-M^2)\,a^2}{M\left[\left(1+a\mathbf{r}\dfrac{da}{d\mathbf{r}}\right)\left(-\vec{\mathbf{z}}\cdot\vec{\mathbf{g}}-\dfrac{F_W}{\mathbf{r}}\right)+\left(1+\mathbf{r}\,M^2a\dfrac{da}{dp}\right)\left(\dfrac{a\hat{c}}{M\mathbf{r}}-\dfrac{a^2}{A}\dfrac{dA}{dz}\right)\right]} \tag{5.b}$$

In particular, if there is no conduit erosion ($\hat{c}=0$) and the conduit has a constant section ($dA/dz=0$), the equations simplify to:

$$\frac{\partial p}{\partial M} = \frac{-\,\mathbf{r}\,a^2}{M\left(1+a\mathbf{r}\dfrac{da}{d\mathbf{r}}\right)} \tag{6.a}$$

$$\frac{\partial z}{\partial M} = \frac{(1-M^2)\,a^2}{M\left(1+a\mathbf{r}\dfrac{da}{d\mathbf{r}}\right)\left(-\vec{\mathbf{z}}\cdot\vec{\mathbf{g}}-\dfrac{F_W}{\mathbf{r}}\right)} \tag{6.b}$$

Given the value of pressure at the entrance of the conduit, the set of equations (5) (or its simplified version (6)) are integrated between the initial Mach number at the conduit inlet, M_i, and the Mach number at the conduit exit, M_o, which is set equal to one (**choked flow** condition is assumed). It allows to compute the variables of interest (pressure, velocity, mass flow rate, void fraction, etc.) along the conduit and its temporal variation depending on the chamber geometry.

2.3. Closure equations

Chamber equations (1) and conduit equations (5/6) need to be completed with closure relationships.

2.3.1. State law. A barotropic state law is assumed in the chamber plus conduit system. When pressure is greater than the nucleation pressure it is assumed that magma behaves as an incompressible liquid with a constant density $\mathbf{r}_l$. When pressure is lower than the nucleation pressure the gas exsolves and a liquid-gas mixture forms. Assuming an homogeneous mixture, density is given by

$$\frac{1}{\mathbf{r}(p)} = \frac{x_e(p)}{\mathbf{r}_g(p)} + \frac{1-x_e(p)-x_c}{\mathbf{r}_l} + \frac{x_c}{\mathbf{r}_c} \tag{7}$$

where $\mathbf{r}_g(p)$, $\mathbf{r}_l$, and $\mathbf{r}_c$ are, respectively, the gas, the liquid, and the crystal densities, $x_e(p)$ is the exsolved gas mass fraction, and x_c is the crystal mass fraction. The gas phase is assumed to follow a perfect gas law, i.e.

$$r_g = \frac{P}{R_g T} \tag{8}$$

where R_g is the gas constant and T is the (constant) temperature. The liquid component of the mixture and the crystals are assumed incompressible. On the other hand, the exsolved gas mass fraction is given by

$$x_e(p) = \frac{x_{tot} - x_d(p)}{1 - x_d(p)}(1 - x_c) \tag{9}$$

where x_{tot} is the initial total gas mass fraction and $x_d(p)$ is the dissolved gas mass fraction, both computed with respect to the crystal-free magma.

2.3.2. Solubility law. CPIUC contemplates two possibilities for the solubility law:

    1. A generic Henry law.

$$x_d(p) = s\,p^n \tag{10}$$

where the exponent n and the constant s are assumed to be independent of pressure and can be determined as a function of magma composition and temperature.

    2. Solubility model proposed in [Zhang, 1999], where:

$$x_d(p) = 0.01\left[a(T)\sqrt{p} + b(T)\,p + c(T)\sqrt{p^3} \right]$$

$$a(T) = 0.4874 - \frac{608}{T} + \frac{489530}{T^2}$$

$$b(T) = -0.060602 + \frac{135.6}{T} - \frac{69200}{T^2} \tag{11}$$

$$c(T) = 0.00253 - \frac{4.154}{T} + \frac{1509}{T^2}$$

where temperature is here assumed constant and pressure is expressed in MPa.

2.3.3. Friction coefficient. The friction coefficient F_W takes into account the pressure drop along the conduit due to the viscous nature of the magmatic mixture. By definition, F_W is the pressure drop due to the friction with the conduit walls. Above the fragmentation level the friction coefficient can be neglected due to the low viscosity of the gas-droplet mixture (orders of magnitude lower that that of the bubbly mixture). Below the fragmentation level the pressure

drop is approximated, depending on the geometry of the conduit, to that of an incompressible laminar flow without body forces (see Appendix A). Possibilities are:

    1. **Circular cross section** conduit of radius r

$$F_w = -\frac{8\,u\,\boldsymbol{m}}{r^2} \tag{12}$$

where $\boldsymbol{m}$ is the viscosity (in general it depends on pressure and temperature).

    2. **Fissure** of width w

$$F_w = -\frac{12\,u\,\boldsymbol{m}}{w^2} \tag{13}$$

    3. **Annular conduit** (ring fault) of radius r and width w

$$F_w = -\frac{4\,u\,\boldsymbol{m}}{r\,(\mathrm{r}+\mathrm{w})} \tag{14}$$

For other conduit geometries (specified by the user), a friction coefficient like (12) is always assumed.

Comment: In hydraulics it is usual to define the friction coefficient as:

$$F_w = -\frac{\boldsymbol{r}\,u^2}{2}\frac{\boldsymbol{x}}{A}f \tag{15}$$

where $\boldsymbol{x}$ is the wet perimeter, A is the flow area, and f is a factor that depends on the flow regime. For laminar flows (straight line of the Moody diagram) the factor f takes the form:

$$f = \frac{a}{\mathrm{Re}} = \frac{a\,\boldsymbol{m}}{L\,\boldsymbol{r}\,u} \tag{16}$$

where a is a constant and L is the characteristic length used to evaluate the Reynolds number. Then, substituting (16) into (15) one has:

$$F_w = -\frac{\boldsymbol{m}u}{2}\frac{a\,\boldsymbol{x}}{A\,L} \tag{17}$$

It is easy to check that for a circular conduit of radius R ($A = \boldsymbol{p}\,R^2$, $\boldsymbol{x} = 2\boldsymbol{p}\,R$, $L = 2R$) expression (12) is recovered if $a = 16$.

2.3.4. Viscosity. Viscosity is a parameter extremely dependent on the flow regime. Above the fragmentation level, in the gas-droplet regime, viscosity can be approximated to that of the gas. In contrast, in the bubbly flow regime (below the fragmentation level), one has to deal with magmatic mixtures which are, in general, composed by a liquid, exsolved gas bubbles and

crystals in suspension. All the three components affect the viscosity of the mixture. In consequence, depending on the flow regime one has:

$$m = \begin{cases} m_{gas} & \text{above fragmentation} \\ m_m & \text{below fragmentation} \end{cases}$$ (18)

where m_{gas} is the viscosity of the gas (zero by default, but can be set by the user) and m_m is the viscosity of the mixture.

1. Viscosity of the mixture m_m. Two possibilities are contemplated in this version of the code:

1.1. Viscosity independent of the gas bubbles $m_m = m_{lc}$, where m_{lc} is the viscosity of the melt (liquid plus crystals).

1.2. Gas bubbles affect mixture viscosity through the Einstein relation [Ishii and Zuber, 1979]

$$m_m = \frac{m_{lc}}{1-a}$$ (19)

where a is the volumetric gas fraction.

2. Viscosity of the melt (liquid + crystals) m_{lc}. Two possibilities are contemplated.

2.1. Viscosity of the melt independent of crystals $m_{lc} = m_l$, where m_l is the viscosity of the liquid.

2.2. Crystals affect the viscosity of the melt through the Roscoe-Einstein relation [Marsh, 1981]

$$m_{lc} = m_l \left(1 - 1.67 \frac{x_c r_l}{x_c r_l + (1-x_c) r_c} \right)^{-2.5}$$ (20)

where r_c and x_c are, respectively, the density of the crystals and the crystal mass fraction.

3. Viscosity of the liquid m_l. Several possibilities are contemplated in CPIUC. In general, liquid viscosity may depend on temperature and pressure (through the solubility law).

3.1. Constant viscosity m_l.

3.2. Solubility model proposed in [Jaupart&Allegre, 1991]

$$\log_{10} m_l = 7.5 - 50 \, x_d(p)$$ (21)

3.3. Solubility model proposed in [Hess&Dingwell, 1996] (Mono Crater rhyolite)

$$\log_{10} m_l = -3.545 + 0.833 \log_{10}(100 \, x_d(p)) + \frac{9601 - 2368 \, \log_{10}(100 x_d(p))}{T - 195.7 - 32.25 \, \log_{10}(100 x_d(p))}$$ (22)

3.4. Solubility model proposed in [Giordano&Dingwell, 2003] (Etna basalt)

$$\log_{10} \boldsymbol{m}_l = -4.643 + \frac{5812.44 - 42704\, x_d(p)}{T - 499.31 + 28.74 \log_{10}\left(100\, x_d(p)\right)} \tag{23}$$

2.3.5. Fragmentation criteria. Magmatic mixture is assumed to fragment when the gas volumetric fraction

$$\boldsymbol{a} = \frac{x_e(p)\,\boldsymbol{r}}{\boldsymbol{r}_g} \tag{24}$$

reaches a critical value that can be set by the user (typically close to 0.75).

2.3.6. The conduit erosion coefficient. The current version of CPIUC does not contemplate conduit erosion (i.e. $\hat{c} = 0$). However, it is expected to incorporate different erosion models nearly in the future.

2.4 Limitations of the model

The main limitations in the current version of CPIUC are

• Handles only volatile-driven eruptions, i.e. eruptions where the critical chamber overpressure responsible for triggering the eruption is reached by exsolution of volatiles during a fractional crystallization process. In consequence, given a chamber depth, the amount of volatiles must be high enough to ensure that the chamber top is vesiculated at the initial pressure value. Future versions will also contemplate the possibility of eruptions triggered by injection of fresh magma into the chamber.

• It is worth noting that the model is able to consider an homogeneous chamber as well as a chamber vertically stratified in volatiles. For stratified chambers, it is assumed that no mixing occurs during the eruption and that progressively deeper portions of the magma are extracted subsequently.

• Chocked flow condition is assumed at the vent.

3. Running CPIUC

To run a generic simulation called *name*, CPIUC uses an input ASCII file (*name.inp*) that specifies all the definitions and parameters that the code needs to run the problem. Eventually, if the user wants to specify a chamber or a conduit geometry different from those already included by default, a secondary input file (*name.geo*) is also needed. After the run, CPIUC produces four different output files: *name.cha*, *name.co*n, *name.exi* and *name.prs*. This section of the manual describes the contents of these files in detail.

3.1. Installation and launching under Windows

Under a Windows environment, CPIUC runs by means of a batch file (*cpiuc.bat*) that opens a console and launches the executable file (*cpiuc.exe*). To run a generic problem *name* just edit the batch file, change the name of the problem and launch the batch clicking on it (be sure that the path of the exe file in the batch file coincides with the real location of the file *cpiuc.exe* in the hard disk). The only limitation of this procedure is that the batch file must be copied to the same folder that contains the input file. After a simulation, the CPIUC output files are written in a folder called *name-results*.

3.2. CPIUC input files

3.2.1. File *name.inp*

Edit this file using the options available to specify the characteristics of a given simulation. The generic ASCII file for input data is based on a series of blocks. Each block starts with the word BLOCKNAME (it can be CONSTANTS, GEOMETRY, RHEOLOGY, SIMULATION or UNITS) and ends with the word END_BLOCKNAME. Blocks contain a series of sentences that define the input data. Each sentence has a word(s) followed by an option or a value. The structure of the file is quite flexible in the sense that, within each block, a sentence can be placed in any order, it can be written using upper or lower cases, etc. If a sentence is omitted in the input file, a default value is assumed. An example of input file is given in **Table 1**. This section describes in detail the syntax and the meaning of each sentence block by block.

NOTE: All the units must be expressed in SI.

? **Constants block**. This block contains information about constants that usually are not changed. In fact, this block can be omitted from the input file since simulations will be normally performed using standard values.

	Sentence	Options
1	UNIVERSAL GAS CONSTANT	*value*
2	GRAVITY ACCELERATION	*value*

? **Geometry block**. This block contains information concerning geometry.

	Sentence	Options
1	CHAMBER GEOMETRY	External
		Rectangular HEIGHT = *value* WIDTH = *value*
		Elliptical HEIGHT = *value* WIDTH = *value*
		Cylindrical HEIGHT = *value* WIDTH = *value*
2	CONDUIT GEOMETRY	External
		Cylindrical DIAMETER = *value*
		Funnel DIAM_TOP = *value* DIAM_BOT = *value*
		Fissure LENGTH = *value* WIDTH = *value*
		Annular DIAMETER = *value* WIDTH = *value*
3	CONDUIT LENGTH	*value*
4	CONDUIT INCLINATION	*value*
5	CONDUIT EROSION MODEL	None

1. Chamber geometry. It can be rectangular, cylindrical or elliptical. In any case, the chamber height (maximum vertical extension of the chamber) and the chamber width (maximum horizontal extension) must be specified. Alternatively, an external user-defined geometry can be proportioned in the file *name.geo*.

2. Conduit geometry. It can be cylindrical, funnel, fissure or annular (ring-fault). For funnel shaped conduits, the diameters at the top and at the bottom must be specified. If the diameter at the top is greater than the diameter at the bottom (divergent conduits) the program may fail because the current version does not handle the formation of shock waves. As in the chamber geometry, an external user-defined conduit can be also proportioned in the file *name.geo*.

3. Conduit length. Length of the conduit. Note that, in general, it does not coincide with the depth of the magma chamber due to the conduit inclination (length is given along the conduit flow direction Z, whereas depth is given along the vertical direction Y).

4. Conduit inclination. Inclination of the conduit with respect to the vertical .It is given in deg.

? **Rheology block**. This block contains information concerning rheology and other physical properties that appear in the closure equations.

	Sentence	Options
1	GAS TYPE	H2O
		CO2
2	TEMPERATURE	*value*
3	DENSITY LIQUID MAGMA	*value*
4	DENSITY CRYSTALS	*value*
5	VISCOSITY MODEL	Constant VALUE = *value*
		Jaupart
		Hess
		Giordano
6	EFFECT OF CRYSTALS	None
		Roscoe-Einstein
7	EFFECT OF BUBBLES	None
		Einstein
8	GAS VISCOSITY	*valu*e
9	EXSOLUTION MODEL	Henry S = *value* N = *value*
		Zhang
10	FRICTION COEFFICIENT	None
		Poiseulle
11	FRAGMENTATION	voidfraction VALUE = *value*
12	CRYSTAL MASS FRACTION	*value*
13	TOTAL GAS MASS FRACT AT TOP	*value*
14	TOTAL GAS MASS FRACT AT BOTTOM	*value*

1. Gas type. Type of volatile. It can be water or carbon dioxide.

5, 6 and 7. Viscosity model and effect of gas bubbles and crystals on viscosity. See section 2.3.4.

8. Gas viscosity. Viscosity above fragmentation. Default value is 0.

9. Exsolution model. See section 2.3.2.

10. Friction coefficient. Specifies the way to compute the viscous contribution in the momentum transport equation. If None, friction is neglected. If Poiseulle, frictional effects are computed depending on the geometry of the conduit as explained in section 2.3.3 and in Appendix A.

11. Fragmentation. Only the void fraction criteria is present in this version. Specifies the void fraction threshold (see section 2.3.5).

13 and 14. Gas mass fraction at the chamber top and bottom. Defines the mass fraction of gas (dissolved plus exsolved) present in the chamber.

? **Simulation block**. This block contains information concerning the simulation.

	Sentence	Options
1	INITIAL CHAMBER TOP PRESSURE	*value*
2	FINAL CHAMBER TOP PRESSURE	*value*
3	NUMBER OF PRESSURE STEPS	*value*
4	NUMBER OF PRINT STEPS	*value*
5	SEARCH INLET VELOCITIES	VMIN = *value* VMAX = *value*
6	SCREEN PRINT	Exit
		Conduit
		Chamber
		Pressure

1 and 2. Initial and final chamber top pressures. During the eruption, magma empties and the pressure at the chamber top decreases. The code begins its computations from the initial pressure and decreases pressure until the final specified value is reached. To have physical meaning, the initial pressure should be close to the lithostatic plus a value high enough to produce the opening of the conduit, whereas the final pressure should be the maximum underpressure that the conduit/chamber walls can hold before collapsing (conduit closure or caldera collapse initiation). In this version of the code, the pressure at the chamber top cannot be greater than the nucleation pressure because it handles only volatile-driven explosive eruptions. If a greater initial pressure is supplied, the program stops and an error message is issued.

3. Number of pressure steps. Number of pressure drop intervals. The time associated with the eruption is computed by integrating the ratio between the mass decrease in a given interval and the mass eruption rate. Since the integration is performed using a trapezoidal rule in each sub-interval, a too small number of integration steps could, in principle, introduce some error in the computation of time. However, in practise, this error is small even for few steps (10 or so).

4. Number of print steps. Number of printed values for each pressure step. For instance, if it is 10, the program writes (in the output files) the main variables at 10 different equally-spaced chamber and conduit points. This is done for each pressure drop interval (for each time), so that the final number of records is 2×number of print×number pressure drop steps.

5. Search inlet velocities. These are a numerical range of velocities needed by the shooting algorithm to find magma velocity at the conduit inlet. Normally, the range 0.01-100 defined in the example of Table 1 covers most real systems. However, try different values if the code cannot find the solution.

6. Screen print. Dumps on the screen console the contents of the selected output file.

• **Units block**. This block contains information concerning the units used in the output files (units in the input file must be always given in SI). The only purpose is to facilitate the output using a "more convenient" system.

	Sentence	Options
1	TIME	Sec
		Min
		Hou
		Day
2	PRESSURE	Pa
		MPa
		Adimensional
3	FLUX	Byzero
		Bysix
		Byseven
		Byeight
		Bynine
		Byten
4	MASS	Kg
		Fraction
5	VELOCITY	ms
		Kmh
6	DISTANCE	m
		Km
		Adimensional

1. Time. Gives the time is seconds, minutes, hours or days according to the option chosen.

2. Pressure. Express output pressures in Pa or MPa. Alternatively, if the option adimensional is used, pressure is divided by the initial pressure at the chamber top.

3. Flux. Divides the mass flow rate by ten to the six, seven, eight, nine or ten.

4. Mass. Gives the erupted mass and the remaining mass in Kg or as a fraction of the initial chamber mass.

5. Velocity. Gives the velocity in m/s of Km/h.

6. Distance. Express output distances in m or Km. Alternatively, if the option adimensional is used, distance is normalized to the conduit length.

```
CPIUC_DATA
 CONSTANTS
   UNIVERSAL GAS CONSTANT  8314
   GRAVITY ACCELERATION     9.81
 END_CONSTANTS
 GEOMETRY
   CHAMBER GEOMETRY  rectangular  HEIGHT 3000  WIDTH 2000
   CONDUIT GEOMETRY   cylindrical   DIAMETER 100
   CONDUIT LENGTH  3000
   CONDUIT INCLINATION   0
   CONDUIT EROSION MODEL  none
 END_GEOMETRY
 RHEOLOGY
   GAS TYPE   H2O
   TEMPERATURE   1273.
   DENSITY LIQUID MAGMA  2400.
   DENSITY CRYSTALS   3000.
   VISCOSITY MODEL    constant  VALUE 1e6
   EFFECT OF CRYSTALS  Roscoe-Einstein
   EFFECT OF BUBBLES     Einstein
   GAS VISCOSITY  1e-5
   EXSOLUTION MODEL  Henry   s = 4.11e-6  n = 0.5
   FRICTION COEFFICIENT  Poiseuille
   FRAGMENTATION    voidfraction VALUE = 0.75
   CRYSTAL MASS FRACTION  0.30
   TOTAL GAS MASS FRACTION AT TOP       0.05
   TOTAL GAS MASS FRACTION AT BOTTOM  0.05
 END_RHEOLOGY
 SIMULATION
   INITIAL CHAMBER TOP PRESSURE   83.6e6
   FINAL  CHAMBER TOP PRESSURE    38.6e6
   NUMBER OF PRESSURE STEPS  5
   NUMBER OF PRINT   STEPS     10
   SEARCH CONDUIT INLET VELOCITIES  VMIN = 0.01 VMAX = 100.
   SCREEN PRINT   exit
 END_SIMULATION
 UNITS
   TIME   hou
   PRESSURE  MPa
   FLUX  byzero
   MASS  fraction
   VELOCITY  ms
   DISTANCE  Km
 END_UNITS
END_CPIUC_DATA
```

Table 1. Example of file *name.inp*

3.2.2. File *name.geo*

This file is necessary only when the user wants to specify the conduit geometry, the chamber geometry or both. It is only used when the option external is marked in the chamber and/or conduit geometry sentences of the GEOMETRY block of the *name.inp* input file. To specify a conduit geometry just give the section of the conduit (in m^2) at different distances Z. The origin of coordinates is at the conduit entrance. Analogously, to specify a chamber geometry, give the chamber section (in m^2) at different depths Y. The origin of coordinates is at the chamber bottom. In both cases the number of records is arbitrary, but they must be ordered following increasing values of Z (of Y). If the chamber/conduit have irregular geometries, the greater the number of records, the greater the precision of computations. **Table** 2 gives an example of file *name.geo* for a cylindrical conduit of length 3Km and radius 10m (section of constant area $314m^2$) and a rectangular chamber of height 2Km and width 3Km (section of constant area 9e6 m^2).

```
CONDUIT
      0        314
      3000     314
END_CONDUIT
CHAMBER
      0        9e6
      2000     9e6
END_CHAMBER
```

Table 2. Example of file *name.geo* (this example is just for illustrative purposes. Clearly it is not necessary to use this file for such a simple geometries).

3.3 CPIUC output files

After a run, CPIUC produces four different output files:

   name.cha. Contains information concerning the chamber. The variable Y is the distance from the chamber bottom measured upwards, whereas depth is the distance from the surface.

   name.con. Contains information concerning the conduit. The variable Z is the distance from the conduit inlet measured upwards, whereas depth is the distance from the surface (they do not necessarily coincide for inclined conduits). Since in the conduit the independent variable is the Mach number, this variable is equally spaced between records. Values at fixed points need to be interpolated from these data.

   name.exi. Contains information at the conduit exit (vent).

name.prs. Contains the pressure distribution along the chamber and the conduit. This can be used to supply the time-dependent boundary conditions to the stress-strain codes.

4. References

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Appendix A. The friction coefficient

The pressure drop due to wall friction F_w is approximated to that of a Newtonian, incompressible, steady, laminar flow without body forces. In this case, the governing equations (Navier-Stokes equations) read
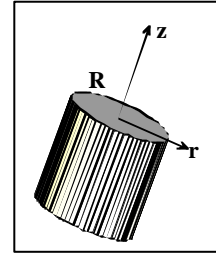
$$\nabla \cdot \mathbf{u} = 0 \tag{A.1a}$$

$$\mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \boldsymbol{m} \Delta \mathbf{u} \tag{A.1b}$$

Consider the following particular cases:

1. Flow in a circular cross-section conduit of radius R.

Choose cylindrical coordinates $(r, \boldsymbol{q}, z)$ with the z-coordinate along the axis of the conduit. By symmetry considerations the velocity field must have the form $u = (u_r, u_{\boldsymbol{q}}, u_z) = (0, 0, u_z(r))$. This velocity field automatically verifies (A.1a) (is divergence free). Substituting in (A.1b) gives, for each component of the vectorial equation

$$\frac{\partial p}{\partial r} = 0 \tag{A.2a}$$

$$-\frac{\partial p}{\partial z} + \frac{\boldsymbol{m}}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u_z}{\partial r} \right) = 0 \tag{A.2b}$$

to solve with the boundary conditions

$$u_z = 0 \quad \text{at} \quad r = R \quad (\text{no-slip}) \tag{A.3a}$$

$$\frac{\partial u_z}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad (\text{symmetry}) \tag{A.3b}$$

Integrating (A.2b) from $r = 0$ to $r = R$ and using (A.2a) and the boundary conditions (A.3) one has

$$u_z = \frac{1}{4\boldsymbol{m}}\left(\frac{\partial p}{\partial z} \right)\left(r^2 - R^2 \right) \tag{A.4}$$

The **flow rate** along a section of the conduit is then

$$Q = \int_A u_z \, dA = 2\boldsymbol{p} \int_0^R u_z(r)\, r \, dr = \frac{\boldsymbol{p} \, R^4}{8\boldsymbol{m}}\left(-\frac{\partial p}{\partial z} \right) \tag{A.5}$$

which is the well-known **Hagen-Poisson law**. The **averaged velocity** $\hat{u}$ is the ratio between the flow rate and the section $A = p\,R^2$

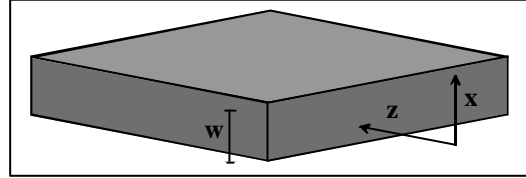$$\hat{u} = \frac{R^2}{8m}\left(-\frac{\partial p}{\partial z}\right) \tag{A.6}$$

so that, finally, the pressure drop per unit of conduit length is

$$\frac{\partial p}{\partial z} = -\frac{8m\hat{u}}{R^2} \tag{A.7}$$

which is the expression used in (12) for the friction coefficient.

2. Flow along two parallel plates separated by a distance w

Choose Cartesian coordinates (x, y, z) with the z-coordinate along the axis of flow. By symmetry considerations the velocity field must have the form $u = (u_x, u_y, u_z) = (0, 0, u_z(x))$. This velocity field automatically verifies (A.1a) (is divergence free). Substituting in (A.1b) gives, for each component of the vectorial equation

$$\frac{\partial p}{\partial x} = 0 \tag{A.8a}$$

$$-\frac{\partial p}{\partial x} + m\frac{\partial^2 u_z}{\partial x^2} = 0 \tag{A.8b}$$

to solve with the boundary conditions

$$u_z = 0 \quad \text{at} \quad x = 0 \ (\text{no - slip}) \tag{A.9a}$$

$$u_z = 0 \quad \text{at} \quad x = w \ (\text{no - slip}) \tag{A.9b}$$

Integrating (A.8b) from $x = 0$ to $x = w$ and using (A.8a) and the boundary conditions (A.9) one has

$$u_z = \frac{w^2}{2m}\left(\frac{\partial p}{\partial z}\right)\left[\left(\frac{x}{w}\right)^2 - \frac{x}{w}\right] \tag{A.10}$$

The **flow rate** (per unit of length y) along a section perpendicular to the planes is then

$$Q = \int_0^w u_z(x)\,dx = \frac{w^3}{12\boldsymbol{m}}\left(-\frac{\partial p}{\partial z}\right) \tag{A.11}$$

The **averaged velocity** $\hat{u}$ is the ratio between the flow rate Q and the length w

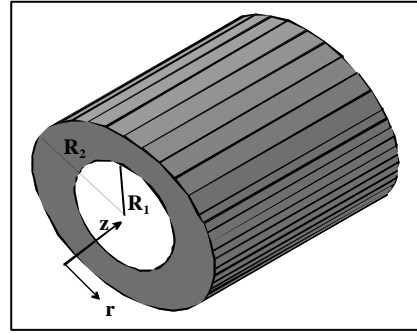$$\hat{u} = \frac{w^2}{12\boldsymbol{m}}\left(-\frac{\partial p}{\partial z}\right) \tag{A.12}$$

so that, finally, the pressure drop per unit of length is

$$\frac{\partial p}{\partial z} = -\frac{12\boldsymbol{m}\hat{u}}{w^2} \tag{A.13}$$

which is the expression used in (13) for the friction coefficient.

3. Annular conduit of internal radius R_1 and external radius R_2.

Choose cylindrical coordinates $(r, \boldsymbol{q}, z)$ with the z-coordinate along the axis of the conduit. Again, by symmetry considerations the velocity field must have the form $u = (u_r, u_{\boldsymbol{q}}, u_z) = (0, 0, u_z(r))$. This velocity field automatically verifies (A.1a) (is divergence free). Substituting in (A.1b) gives, for each component of the vectorial equation

$$\frac{\partial p}{\partial r} = 0 \tag{A.14a}$$

$$-\frac{\partial p}{\partial z} + \frac{\boldsymbol{m}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) = 0 \tag{A.14b}$$

to solve with the boundary conditions

$$u_z = 0 \quad \text{at} \ \ r = R_1 \ \ (\text{no - slip}) \tag{A.15a}$$

$$u_z = 0 \quad \text{at} \ \ r = R_2 \ \ (\text{no - slip}) \tag{A.15b}$$

Integrating (A.14b) from $r = R_1$ to $r = R_2$ and using (A.14a) and the boundary conditions (A.15) one has

$$u_z = \frac{1}{4\boldsymbol{m}} \frac{\left(\dfrac{\partial p}{\partial z}\right)}{\ln\left(\dfrac{R_2}{R_1}\right)} \left[\ln\left(\frac{R_2}{R_1}\right) r^2 + R_2^2 \ln\left(\frac{R_1}{r}\right) - R_1^2 \ln\left(\frac{R_2}{r}\right)\right]$$ (A.16)

The **flow rate** along a section of the conduit can be obtained after some tedious algebra as

$$Q = \int_A u_z \, dA = 2\boldsymbol{p} \int_{R_1}^{R_2} u_z(r)\, r\, dr = \frac{\boldsymbol{p}}{8\boldsymbol{m}}\left(-\frac{\partial p}{\partial z}\right)\left(R_2^4 - R_1^4\right)$$ (A.17)

The **averaged velocity** $\hat{u}$ is the ratio between the flow rate Q and the section $A = \boldsymbol{p}\,(R_2^2 - R_1^2)$

$$\hat{u} = \frac{1}{8\boldsymbol{m}}\left(-\frac{\partial p}{\partial z}\right)\left(R_1^2 + R_2^2\right)$$ (A.18)

so that, finally, the pressure drop per unit of conduit length is

$$\frac{\partial p}{\partial z} = -\frac{8\boldsymbol{m}\hat{u}}{R_1^2 + R_2^2}$$ (A.19)

In those particular cases where $R_2 = R_1 + w$ with $w \ll R_1$, the above expression can be approximated as

$$\frac{\partial p}{\partial z} = -\frac{4\boldsymbol{m}\hat{u}}{R_1\,(R_1 + w)}$$ (A.19)

which is the expression used in (14).